TABLE 5.—*Sun spot and magnetic epochs, 28-month period*

Sun spot		Magnetic		Sun spot		Magnetic	
Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
1837.0	1838.2	1836.5	1837.5	1880.2	1881.0	1879.2	1880.2
1839.5	1840.5	1838.7	1839.7	1882.0	1883.0	1881.7	1882.7
1841.2	1842.0	1841.0	1842.2	1884.0	1884.7	1884.0	1885.0
1842.7	1844.0	1843.2	1844.2	1885.7	1887.0	1885.7	1887.0
1845.2	1845.7	1845.2	1845.7	1888.5	1890.2	1888.0	1889.5
1846.7	1847.2	1846.7	1847.2	1891.7	1893.0	1891.2	1892.5
1848.5	1849.5	1848.7	1849.5	1893.7	1895.0	1893.5	1894.5
1850.0	1851.5	1850.5	1851.5	1896.0	1897.2	1895.7	1897.7
1852.2	1853.7	1852.5	1853.5	1898.2	1899.7	1898.7	1900.5
1854.7	1856.2	1854.7	1856.2	1900.7	1902.7	1901.5	1902.5
1857.7	1858.7	1857.5	1858.7	1903.7	1904.7	1904.7	1906.5
1860.0	1861.5	1860.0	1861.5	1906.5	1908.0	1907.5	1909.5
1862.5	1863.7	1863.0	1864.2	1909.2	1911.0	1910.2	1912.0
1865.2	1867.0	1865.5	1867.2	1912.7	1914.2	1913.2	1914.2
1868.7	1869.7	1868.7	1869.7	1915.5	1916.7	1915.7	1916.5
1870.5	1871.7	1870.7	1871.7	1917.5	1918.2	1917.7	1919.2
1872.7	1873.7	1872.7	1873.5	1919.7	1920.7	1920.0	1921.5
1874.7	1876.0	1874.5	1875.5	1922.2	1923.7	1922.7	1924.0
1877.2	1879.0	1876.7	1878.0	1925.7		1925.7	

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THE RATE OF DECAY OF ATMOSPHERIC EDDIES

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INTRODUCTION

The way in which eddies are produced and the rate at which their energy is dissipated in actual fluids is a matter of considerable importance, both from a practical standpoint as a measure of the effect of viscosity on atmospheric motions in its relation to aerodynamical and meteorological problems, and also for theoretical reasons as a means of furnishing a desirable basis for, and a check upon the mathematical investigations regarding the motion of perfect fluids; that is, fluids in which similar motions may occur and which may have all of the other properties of actual fluids, but which we may imagine to be entirely devoid of internal friction, or viscosity.

In the present paper it is attempted to investigate the dynamical nature of individual eddies, their growth and rate of decay, etc., rather than to make a statistical study of the combined behavior and average effect of a large number of them, which has already been so admirably done in the work of a number of other students of this subject.

The theory is more or less restricted to eddy motions taking place at a fixed altitude and with supposedly the least possible exchange of thermal and gravitational energy, and therefore only a slight reference will be found to that part of the problem relating to the motion of eddies which may, over a given time, be increasing, or decreasing with height relative to any fixed levels.

OBSERVATIONS ON THE DISSIPATION OF EDDY MOTION

Selecting certain days when the air was very clear and quiet, some attempts were made to determine the rate of decay of eddies set up and following in the rear of railway trains, motor cars, and other vehicles moving on, or near, the earth's surface.

The methods employed in this investigation were necessarily of such nature as to afford but little opportunity for making any very exact measurements in the process of collecting the required data, since there were no means available for making an accurate quantitative study of the velocities and time intervals involved, as is otherwise the case where the experiments are under the complete control of the observer, and therefore the results obtained in the present work can hardly be considered as being much more than rough approximations made in the absence of any better and more exact facilities for experiment.

The usual procedure in this investigation was to stand as near the track as possible until the train had passed by and then to release bits of paper or other light material, and observe the lapse of time and how far the train traveled before the eddies caused by its motion had become entirely dissipated.

The distance which the train moved during the interval could not be very accurately estimated in most instances, but knowing the usual speed of the particular train passing at that time and by taking note of certain objects along the right of way which were apparently about even with the rear of the train by the time the eddies, as indicated by the motion of the bits of paper, had completely died away; then by measuring this distance it was possible to obtain a rough estimate of how far the train had traveled during that period of time.

It was found that in the case of a fast passenger train moving at a speed of around 25 meters per second (or about 56 miles per hour), the eddies formed by its motion seemed to be dissipated in about 12 seconds; that is, the air over the tracks had settled into its usual quiet state again after the rear of the train had reached a point about 1,000 feet away.

At 12.5 meters per second the distance traveled was approximately 375 feet and the time required for the air to become quiet again was about 9 seconds, and at one-half the above-mentioned velocity, or about 6.25 meters per second (14 miles per hour), the distance was around 125 feet and the time 6 seconds.

This seems to indicate that the rate of decay is quite rapid at the relatively higher speeds, but the motion of the eddies becomes much more persistent for equal time intervals at lower velocities.

In the curve Figure 1 is shown the relation between the speed of the train and the time rate of decay of the eddies produced by it. The part of the curve indicated by the dotted line is that which it is assumed would be obtained if the speed should be doubled three or four times beyond that at which the actual observations were made.

It is found that the velocity (that is, the apparent velocity within the eddies as determined with reference to the relative speed between the body and the air) falls 50 per cent for equal time intervals from whatever is considered the initial value given them at the particular instant when they are first developed; and the curve becomes asymptotic to the vertical ordinate at something less than 770 miles per hour, which seems to indicate that at velocities exceeding that of sound the eddies themselves cease to acquire any additional velocity as the speed of the body increases beyond this point, and it is quite probable that the disturbance created by the motion of the body changes to one of a fundamentally different nature after reaching this value.

It is of considerable interest to study the motion, or flow pattern, of the air along the side and in the rear of a fast-moving train.

Stepping on to the middle of the track and releasing a handful of small bits of paper immediately after the last car had passed by it was found that these bits of paper at first flew away after the train with almost the same velocity as the train itself. Their motion was practically always slightly downward between the rails, then outward toward each side of the track and then upward, sometimes higher than the car. Their path of motion, in fact, seemed to follow the turns of a long helix, one on each side of the track and revolving in opposite directions. Since the motion near the ground seemed to be always outward and then upward from each side of the track, there must be a corresponding inflow over the top of the train and downward over the middle of the track.

Figure 2 is a two-dimensional diagram of the flow pattern of the eddies produced by a moving train. This may be a rather idealized picture of the circulation, because it is not certainly known whether or not the paths of motion are nearly circular about two imaginary central lines parallel to the direction of the train. The distribution of velocity and the pattern of the streamlines in the actual motion may, of course, vary considerably from one instant to another on account of the variations in air movement, but for the purpose of analysis we can imagine the motion to be steady at any one instant and the paths of motion of the air in each eddy to follow the circular pattern shown in the diagram, and at the same time there is also a motion of translation in the same direction as that of the train, so that the actual path for the fluid elements of each ring is that of a helix.

The movement of the air disturbed by the train could easily be detected at a distance of about 4 meters on either side of the middle of the track when there was no cross current of wind and the extent of this disturbance seemed to be always of the same dimensions no matter whether the speed of the train was 15 or 60 miles per hour.

Almost every portion of fluid in whatever path of motion it may follow always has a certain cause, somewhere, for its taking that particular course, and the most complex motions may have definite laws which can explain their behavior when once we understand them.

The twin helices of air motion following immediately in the rear of most bodies, and especially those of rather abrupt form, are quite well defined and unmistakable. They may be easily observed from the rear window of an automobile when moving at a moderate speed along a dusty road, or they can be distinguished in the whirls of dust and in the exhaust smoke from a car a short distance ahead. The scatter of leaves, bits of paper, etc., from under a passing automobile or street car will give the same indication, and perhaps another more ideal place for observing this systematic type of turbulent motion would be the rear platform of a railway coach, especially from the parlor-observation car at the end of the train.

The most important, and at the same time the most difficult, problem connected with observations on the decay of eddies set up by moving bodies is that of determining the initial tangential velocities within the eddy itself, as the rate of decay was found to depend only on this circulation and was not in any way directly related to the speed of the train or other body from which the eddies were produced, or to the forward speed imparted to the air in the following flow.

In fact, by the time the revolving motion around the helical axis has died out, the forward motion of the whole turbulent wake of fluid will also have been dissipated.

The only available means of determining these velocities was to measure the distance from the point where the bits of paper were released to the point where they were deposited along the right of way by the outward moving portion of the circulation near the ground. Then, since the path followed by these particles with reference to a plane normal to the helical axis formed about one-fourth of the outer circumference of the eddy, and knowing the approximate speed of the train, it was possible to obtain a rough estimate of the tangential velocity of this outer ring of fluid by taking the ratio between this distance and the distance which the train moved during one second of time.

Thus in one series of observations the mean distance from a point in the middle of the track where the bits of paper were released to the point where they were deposited by the outward moving air current was about 61.3 feet, in the case of an express train moving at a speed in the neighborhood of 25 m. p. s. Then, since the radius of the outer part of the eddy was found to be approximately 2 meters, the time taken for the particles to move over one-fourth of their path in describing the circumference of this outer ring was about three-fourths of one second.

Another series of observations made under about the same atmospheric conditions, but in the case of a train moving only at about 41 feet per second, gave a distance slightly in excess of 40.5 feet, and therefore the time taken to move around one-fourth of the circumference was also one second.

An exact formula was worked out for the purpose of making a quantitative analysis of these motions and for the determination of the velocities therein, and this method together with the results obtained from it will be explained in another section of this paper.

The data given above are only very rough approximations and no claim is made in regard to their being entirely accurate, but since they represent the average values derived from a large number of observations it is hoped that they may be accepted for the present, and perhaps with more precise methods of experiment the exact relations can be determined later.

Shortly after making the first preliminary attempts to measure the tangential velocities within eddies produced by railway trains it was noticed that these velocities were not directly proportional to the relative speed, and in fact they did not seem to bear any definite relation whatever to changes in the relative motion between the body and the air.

The eddy motion in the rear of one train moving at twice the speed of another did not appear to have anyways near the increase in velocity that it should have in proportion to the difference in the relative speed.

This was rather a puzzling situation until it finally became apparent that as the air in the layer next to the surface of the car was dragged along with it in its motion, the velocities in the eddies produced in the rear of the train depended upon the rate at which this marginal layer escaped from the forward drag of the cars, or upon the rate at which the fluid elements were shed backward from the rear terminus of the boundary layer.

In Figure 3 it is attempted to illustrate what actually takes place when an air current passes along a stationary wall which suddenly breaks off at an angle. It is assumed here for convenience that the motion is steady and two dimensional.

The fluid immediately in contact with the wall is at rest relative to it and the transition from the stationary to the moving elements occurs in this marginal layer near the surface. Then the velocity in each succeeding layer outward in a direction normal to the surface becomes greater as this distance increases until it reaches that of the undisturbed flow.

The fluid layers next to the boundary probably contain elements moving with very high vorticity to compensate for the transition from the moving to the stationary air, but the details of this action can be ignored for the present, as what concerns us most now is the relative velocity between the body and the layer in motion immediately outside the boundary. As this layer moves along and reaches an abrupt corner of the body it meets with the marginal layer in contact with the rear surface and whatever velocity may be permitted it by the compensating actions taking place between this layer and the actual surface will allow a corresponding amount of fluid to be released from that portion having what would otherwise be the same velocity as the body itself.

When a body, such as A in the diagram, moves at any velocity relative to the undisturbed flow F, the motion within the two marginal layers B and D are equal, although they are not of the same thickness, and as the fluid is fed backward it requires a replacement of new elements from these two directions which sets up a circulation around C.

The next succeeding layer outside of the one affected by the boundary, although it has a higher velocity relative to that of the body, is less disturbed in its motion, and the other outer layers are still less affected until at a certain distance the velocity becomes that of the total undisturbed flow.

If the motion of the fluid fed back from the boundary layer was the same at all points of the rear edge of the surface, and if from each point the flow should be always parallel to the general direction of the relative motion between the body and the fluid, as implied by the two-dimensional diagram shown herewith, then the eddy motion would consist of a circulation set up around an axis parallel to the terminal edge of the boundary layer; but if other surfaces should be arranged so as to establish a three-dimensional flow it would be very difficult to provide for all points to have equal vorticity, or for all portions of the boundary layer to be of uniform thickness, and for the fluid elements contained in this layer to have everywhere the same direction and velocity, etc., except in such cases as may occur where a symmetrically shaped body like a cylinder was made to move parallel to its axis through the fluid.

In most cases of three-dimensional motion, however, with bodies of other than circular cross section, there is a tendency for more air to be shed backward from one point than from another as, on account of the unequal distribution of pressure, the direction of flow is not always parallel to that of the relative motion, and therefore because of this convergence, or divergence of the air flow to, or from certain points of the marginal layer a circulation is produced around axes parallel to the general direction of motion which originate at the rear surface of the body and extend back over the path traveled since the beginning of its motion.

This is one of the few types of flow which seem to meet all the conditions for stability, but to this statement a reservation must be added to the effect that this obtains only so long as the oppositely directed centers of rotation are not made to approach too near each other.

As far as could be learned from the observations previously described the two centers of oppositely revolving fluid in this case preserved their identities until the motion was completely dissipated.

There may naturally be some doubt as to whether or not the size of an eddy remains constant over all ranges of relative velocity between a solid body and the surrounding fluid.

As already stated a moving train seemed to distrub the air only to a distance of about 4 meters to either side of the middle of the track which it passed over, regardless of whether it was moving fast or slow. It was found to be quite easy to step into or out of the air movement caused by its motion while the train passed, as the boundary of the disturbance was very clearly defined.

The particular point of interest in this connection, however, outside of determining the extent of the disturbance for different speeds, was that if one stood at a certain distance from the track, the faster the train was moving the further the head end of it was past before the air blast could be felt. For a slow moving train the rush of air along side of it was experienced almost at the same time as the engine passed, but in the case of a fast moving express this rush of air could not be felt at a distance of 4 meters from the middle of the track which it passed over until two or three seconds after the front end of the locomotive was past. Of course when one stood nearer the track the disturbance in the air became apparent much sooner than that, but in no case, providing the surrounding air was quiet at the time, did this disturbance seem to extend much beyond 4 meters from the middle of the track.

It should not be understood from this, however, that eddies formed by the relative motion between a body and the surrounding fluid do not ultimately assume any larger dimensions than those to which they develop near the disturbing object.

It is simply intended to point out here that there is a definite relationship between the size and shape of the body and the size of the eddies which it creates in the fluid at the instant of their maximum development, and this seems to be independent of the relative speed; but this does not mean that there can be no spreading out, or a more or less increase in the size of the eddy under the influence of viscosity after it is once free of the forces by which it was produced.

MATHEMATICAL THEORY

The theory of dissipation of eddies has been investigated mathematically by Taylor (**1**), (**2**), Webb (**3**), and Bateman (**4**), and also by Richardson (**5**) and Fujiwhara (**6**).

Since Taylor was the first to formulate the dynamical principles of this theory and his analysis (1) has formed the basis for most of its subsequent development, a brief résumé of his paper will be given here for the benefit of those who are not already familiar with it, and a few notes will also be added in regard to the more important points in some of the other papers mentioned.

In Taylor's analysis the motion is supposed to be two dimensional. If we let ω be the angular velocity of a ring of fluid of radius r, the tangential velocity $v=\omega r$.

Then if μ is the viscosity and ρ the density, the equation for the motion of any ring is

$$(1)\quad \frac{d}{dr}\left\{2\pi r^2\mu r\frac{d\omega}{dr}\right\}=2\pi r^3\rho\frac{d\omega}{dt}$$

where $t=$time.

Substituting $v=r\omega$ and $\nu=\frac{\mu}{\rho}$, the equation becomes

$$(2)\quad \frac{d^2v}{dr^2}+\frac{1}{r}\frac{dv}{dr}-\frac{v}{r^2}=\frac{1}{\nu}\frac{dv}{dt}$$

By substituting a new variable θ, in the velocity term, the above equation becomes

$$(3)\quad \frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}=\frac{1}{\nu}\frac{d\theta}{dt}$$

which is identical with that for the conductivity of heat

In this case a solution for θ is known to be

$$\theta=\frac{A}{t}e^{-\frac{r^2}{4\nu t}}$$

Hence a solution of (2) is

$$v=\frac{d\theta}{dr}=-\frac{Ar}{2\nu t^2}e^{-\frac{r^2}{4\nu t}}$$

On putting

$$(4)\quad \zeta=\frac{r}{\sqrt{4\nu t}}$$

it will be found that

$$(5)\quad v=Bt^{-\frac{3}{2}}\zeta e^{-\zeta^2}$$

where B is a constant.

Considering an eddy of a type in which the velocity is zero at the center and at infinity; this velocity is a maximum where

$$\zeta=\frac{1}{\sqrt{2}}$$

And if the radius R of the eddy be defined as the radius of the ring of maximum velocity, it will be seen from (4) that

$$(6)\quad \zeta=\frac{1}{\sqrt{2}}=\frac{R}{\sqrt{4\nu t}}$$

Hence $$(7)\quad R=\sqrt{2\nu t}$$

Here t evidently represents the time taken by the eddy to attain a radius R, starting from the condition in which ν and ω are infinite at time $t=0$. This may therefore be called the "age" of the eddy.

We may imagine that an eddy of radius a is formed and that it is of standard form given by equation (5). Then its initial "age" t_0, i. e., the time it would have taken to form itself from a nucleus originally concentrated along the axis is given by (7). It is

$$(8)\quad t_0=\frac{a^2}{2\nu}.$$

Taylor has shown that at some later time when the "age" is t, the velocity will have died down to $\frac{1}{n}$th of its initial value where t is given by

$$\frac{1}{n}=\frac{t^{-\frac{3}{2}}}{t_0^{-\frac{3}{2}}}$$

and therefore $$(9)\quad \frac{t}{t_0}=n^{\frac{2}{3}}$$

Hence from (8) and (9) the time taken for an eddy of initial radius a to die down to $\frac{1}{n}$th of its velocity is

$$(10)\quad t-t_0=t_0\ (n^{\frac{2}{3}}-1)\ \frac{a^2}{2\nu}\ (n^{\frac{2}{3}}-1)$$

Putting $\nu=.14$ and taking the case of an eddy whose initial diameter is about 0.6 cm. corresponding with a cylinder of the same diameter, then $a=0.3$ cm. and $\frac{a^2}{2\nu}=0.32$ second.

Therefore the time taken for an eddy to die down to half its velocity is found by putting $n=2$, in equation (10). It is 0.18, or about one-fifth second.

In Webb's paper both the development and decay of an eddy, under the combined influence of a line source along the axis and viscosity, is investigated mathematically and a solution is obtained of Taylor's equation (2) to which has been added the term $+\frac{2c\nu}{r^2}$, where c is a constant and is connected with the radical velocity by the relation $u=\frac{c}{r}$.

Bateman has obtained a generalization of Taylor's equation which gives a solution of the three-dimensional problem of an eddy with a source or sink along the axis, following the type considered by Webb.

Bateman's paper contains some interesting mathematical relations connected with the theory of this type of motion and points out an analogy between the theory of eddies in a viscous fluid and the Quantum theory of radiation.

He has also obtained another exact solution of the equations of motion of a viscous fluid which gives a result comparable to the well-known principle that a horizontally stratified atmosphere is unstable when the lapse rate of temperature is higher than the adiabatic value.

Lamb (**7**) has given some attention to a problem quite similar to this, in which the fluid motion is supposed to be in circles about an axis, the velocity being a function of the distance r from this axis.

A very good summary as to the general nature of this problem is to be had in a quotation from a part of Taylor's paper in which he explains the dissipation of eddies as probably being due to two main causes; "(a) the action of viscosity between rings of fluid rotating with different angular velocities, and (b) forces due to dynamical causes which tend to reduce the tangential velocity in the eddy. A possible cause of this kind would be the flow inward from the ends of the eddy toward the middle, down the axis of the eddy. This flow would allow the radius of the rotating part to increase and at the same time the velocity would necessarily decrease in order to keep the angular momentum constant.

The action, in fact, would be the opposite of that of a sink; e. g., the waste hole in a bath, where the taking away of fluid from the middle decreases the radii of the rotating rings of fluid and consequently increases their angular velocity.

It is very important that a marked distinction should be made between two different kinds of eddies, or rather between two different stages of their life history. In their first stage, that of growth and development, their motion is produced and maintained by forces external to that of the eddy circulation, such as the dynamical interaction between two currents having different velocities or directions, or between the fluid and a solid body in motion relative to each other; whereas the second stage, or that of decay, the motion is due entirely to the momentum of the fluid mass within the limits of the eddy itself.

In the first stage the field of motion may be built up from one or more nuclear axes, or filaments, and it will continue to be maintained against the retarding action of viscosity so long as the external forces remain operative. This type may be known as a *driven* eddy.

It often happens, however, that these external forces are but momentarily active upon a definite portion of fluid, and following the application of a sudden initial impulse sufficient to start the motion, these forces may as suddenly cease to act, after which the motion is kept up almost entirely by the momentum imparted to the fluid until it is finally dissipated by opposing forces due to the friction between the relatively moving elements of the fluid. This type may be known as a *free* eddy.

Before attempting a quantitative discussion of the results obtained in the present study, it is necessary to make certain fundamental assumptions as to the exact nature of the particular form of fluid motion that is here under consideration.

1. It is assumed that, except within a limited region very near the axis the tangential velocity is inversely proportional to the radius, or in other words, that the product of the circumference of any selected streamline circle and the velocity on this circumference is constant and has the same value for every circle outside of the central nucleus, since the velocity decreases in the same ratio as the circumference increases. The tangential velocity is greatest at the ring forming the boundary between the rotating and nonrotating parts of the eddy, and the velocity of the external field of fluid at an infinite distance from the nucleus is taken to be zero.

In the practically important cases the velocity in the irrotational part reduces to that of the main stream of fluid generating the motion within a certain limited distance of the nucleus.

In view of the foregoing it will be easily understood that the angular momentum of each ring of fluid represented by any streamline denoting the flow around the axis at any given distance from it, has a constant value which is the same for each successive ring between the outer and inner limits of the irrotational field.

2. The retarding force due to the action of viscosity between successive rings also has a constant value at any point within this irrotational field of motion, and is directly proportional to the differences in velocity between each of the different rings. The total retardation within the whole eddy may be considered to have the same value as it would have if it were confined entirely to the circumference of the ring of maximum velocity.

There is then a constant relation between the radius of any ring of fluid outside of the nucleus, the angular momentum, and the amount of retardation due to viscosity, and it is therefore independent of the dimensions of the eddy.

3. The radial velocity is directly proportional to the tangential velocity; and the inflow parallel to, or along the axis, is proportional to the *square* of the tangential velocity.

Note.—In speaking of "rings of fluid" it should be understood that this is merely a convenient expression for describing the flow and that actuelly they must represent such a small radial difference that there can be no relative motion between their inner and outer limits.

It simply makes the analysis easier if we imagine the fluid elements involved in the motion to be separated into rings which decrease in velocity and increase in thickness as their radius increases.

In order to describe more fully the various details as outlined in our first assumption in regard to the distribution of the velocity and angular momentum of a fluid revolving around a central nucleus, it may be stated that we need only to know, aside from the fact that the region outside the nucleus is free from rotation, the location and the diameter of this nucleus and a single number which designates the velocity at a unit distance from it. Instead of this last number, one 2π times as large is usually given, which is termed the "circulation." It is the product of the circumference of any selected streamline circle and the velocity on this circumference. As already mentioned, this product is the same for every circle outside the nucleus, since the velocity decreases in the same ratio as the circumference increases. This "circulation" number therefore defines the magnitude of the nucleus. If the rotation in the nucleus is uniform (like a solid body), then the "circulation" is equal to the product of the rotation times the area of the nucleus, for, if r represents the radius of the nucleus and ω the angular velocity, then the circumferential velocity of the nucleus is $r\omega$, the circumference is $2\pi r$ and the circulation is their product: $2\omega\pi r^2$, which equals the product of the area of the nucleus πr^2 times the rotation 2ω. If the rotation is not uniformly distributed then its mean value must be taken, which multiplied by the area of the nucleus, will then give the circulation about the nucleus.

It is thus seen that it is only necessary to know the location of the rotating parts of the fluid and the amount of their rotation in order to determine all the rest of the data. It may seem strange to many that anything so small as the nucleus of a vortex can exert a determining influence on all the rest of the current. The explanation lies in the assumption that the flow is free from rotation with the exception of the small area of the nucleus. Perhaps this nucleus is often wrongly regarded as the mechanical cause of the vortex field. As a matter of fact, the irrotational flow is produced by pressure, and where there is considerable shearing stress, regions of rotational motion are generated which are variously distributed by the motion of the main current.

It may therefore be said more truthfully that the whole flow generates and distributes the vortex nuclei. In any case, the size and distribution of the nuclei is very closely connected with the motion of the rest of the fluid, so that we may calculate backward from the distribution of the nuclei to the corresponding irrotational flow.

As the motion dies down the neighboring parts of the fluid are likewise gradually set in rotation and themselves form components of the nucleus. Hence the nuclei gradually spread out. The "circulation" about the nucleus is not changed thereby, because we can always measure it at any desired distance from the nucleus, hence at so great a distance that the influence of the viscosity is not felt.

If, however, the nucleus enlarges without increasing the circulation, then the mean rotation of the fluid in the nucleus must have been distributed over the larger area. The law that the "circulation" remains constant in this extension of the nucleus, holds good only so long as two or more nuclei do not merge into one another.

And further, according to this law, the nucleus can not end therefore at any point whatsoever in the fluid. It must either form a closed ring, or spread out to infinity, or end at the edges of the fluid. (8).

The latter condition is always present in practically every case with which we are concerned, since most eddy motions in the atmosphere always take place in the neighborhood of the boundaries formed by solid bodies, or within a reasonable distance of the earth's surface.

It should be mentioned here that what has just been said in regard to the fundamental laws of vortex motion, and their close approximation to actual conditions in the local eddying, or turbulent motions of the atmosphere, can not be applied to any such large scale motions as the migratory cyclone of the temperate latitudes and possibly only with certain modifications to intense tropical cyclones, because the ordinary cyclonic disturbance is of such vast area compared to its vertical extent and there is such a great difference in the temperatures and water vapor content, etc., of the different air streams which go to form part of its general wind system, that there is only a very slight similarity between it and an ordinary eddy, unless these other more complex relations are fully accounted for.

The mean temperature of the air during the period in which the observations on the rate of decay of eddies were made was about 64.5 F., or 18 C.

Now the density ρ of air at this temperature and at about 41 N. latitude and at an altitude of 225 meters above mean sea level is 0.00120 g/cm., or 1.20 kilograms per cubic meter.

The coefficient of viscosity of air varies in relation to its temperature and, according to information furnished by the United States Bureau of Standards, about as good a formula as any for determining this relation is

$$\mu = 0.00018240 - 0.000{,}000{,}493(23 - t^{\circ})$$

Since t° in this case = 18.0 C., then $\mu = 0.000179935$, or practically 0.00018; the kinematic viscosity $\nu = \frac{\mu}{\rho}$ is therefore

$$\nu = \frac{0.00018}{0.00120} = .15.$$

Now, if we consider the inside of the eddy near the axis as a region of uniform vorticity defined by the relation v prop. r, where r is the distance of a point from the central axis and v is the tangential velocity at that point at right angles to r, then if at every point outside of this central nucleus the velocity decreases in the same ratio as the circumference increases, this outer region will have no vorticity and the momentum of the fluid contained within a very thin ring, say, that defining the boundary between the rotational and irrotational portions of the eddy, will be equal to $2\pi r v \rho dr$, where dr is the thickness of the ring; and, further, since the product of the velocity and the circumference is constant for each ring of fluid outside the nucleus (within the limits of the irrotational field of motion), therefore the total angular momentum of the fluid contained in the whole cross-sectional area between the inner and outer limits of the irrotational field will be

$$\mathrm{M} = \int_0^{\infty} 2\pi r^2 v \rho dr$$

Since by our second assumption the retarding action due to friction between the different rings is also constant and has the same value at every point within this region, the time taken for an eddy of this type to die away will therefore depend on the excess of energy due to the momentum of the fluid within the boundaries over the retarding action caused by the viscous drag, or shearing stresses, developed between the different rings. Hence, as a comparative measure

$$(1)\quad \tau = \int_0^{\infty} \frac{2\pi r^2 v \rho dr}{2\pi r^2 \mu dr}$$

As we may cancel those parts of the expression which define the size of the eddy, we have simply

$$(2)\quad \tau = \frac{v\rho}{\mu}$$

Thus the time rate of dissipation depends only on the density and viscosity of the fluid and it is directly proportional to the maximum tangential velocity of the eddy relative to the surrounding medium.

In determining the tangential velocity of the fluid within the irrotational part of the eddy it is, of course, necessary to determine it with reference to some point at a certain radial distance from the center, and it is in this part of the problem that we are concerned with the dimensions of the flow.

It proved to be rather difficult to find a method of making exact calculations of the tangential velocity of the fluid following the helical path of motion described, but after considerable study a fairly satisfactory formula was worked out, which will probably be made clearer by reference to the diagram Figure 4.

In this case supposing dx to be the distance which the train moved in one second, and ds the distance to which the bits of paper were carried during the same interval while they moved to a point approximately one-fourth the way round the circumference of a circle of radius

$r=2$ meters from the point where they were released, then the tangential velocity around the helical axis was found to be given by an expression of the form

$$(3)\quad v=\frac{d\theta}{dt}=\frac{d\theta}{ds}\,(dx-ds)+\frac{d\theta}{1}.$$

Where θ is the distance in meters around that part of the circular path over which the particles moved in unit time and it is related to v by the relationship $\frac{d\theta}{dr}=v$.

For a train moving at the rate of about 28 miles per hour the tangential velocity was found to be something like 3.15 meters per second, so that

$$\frac{v\rho}{\nu}=25.2.$$

Now if the density and other conditions of the fluid should remain the same, but the velocity be varied by a given amount, it is necessary to multiply $\frac{v\rho}{\nu}$ by some constant in order to determine the rate of decay corresponding to the new velocity.

At an initial tangential velocity of around 3.15 meters per second the time required for it to have become entirely dissipated was found to be about 9 seconds.

The coefficient connecting the rate in this case with that of any other velocity, under similar conditions, is therefore

$$(4)\quad c\tau=\frac{25.2}{9}=.3572.$$

Hence the rate of decay for any other eddy in a fluid having the same density and other physical characteristics as that in which these observations were made, but differing in velocity and dimensions only, will be given by

$$(5)\quad \tau=\frac{v\rho}{\nu}\ (.3572).$$

It is not known what may be the value of $c\tau$ in air of a different density, etc., from that in which the above values were obtained. It must necessarily increase with altitude, but its exact relation to density is as yet undetermined.

If this value increases in the same ratio as the density decreases, then the time rate of decay for an eddy starting with a given initial velocity would be the same at all altitudes, providing, of course, the motion of each of its component elements takes place under normal conditions as to temperature and density, etc., pertaining to that altitude at which the motion may occur.

In other words, this value would change if the eddy itself should be moving upward or downward, or if within the field affected by its motion there should be a transport of air from one level to another.

It is very likely that $c\tau$ increases more rapidly than the proportional decrease in density, and at any rate it is probably many times greater at a height of two or three thousand meters than what it is near sea level.

It is also admitted that so far this study has not furnished any information as to what proportion of the dissipation of eddy energy may be due to dynamical causes and what part to viscosity, because, as Taylor has already remarked, the exact distribution of the velocity is not known.

But without this information it may be permissible to assume, however, that the axial and radial motions in eddies of this type are related to the tangential velocities in such manner that the part of the decay caused by the dynamical effect is a function of the decrease in velocity of the fluid revolving around the axis, and if the rate of this decrease is known it might be taken as a measure of the effect due to both causes.

It seems there have been no very recent studies made with a view of computing the actual velocities which occur within eddies of the type in which the dynamical effect would be fully developed. The only information available to me at the present time is that contained in a series of papers by Prof. F. H. Bigelow on "Studies on the vortices in the atmosphere of the earth." (**9**).

I believe the results of his analysis of the motion of air and its component velocities within tornadoes and waterspouts are entirely dependable and sufficient in representing actual conditions for the purpose of determining the relationship between these different velocities, and from a study of Table 35, contained in his analysis of the St. Louis tornado of May, 1896, it was found, at least for that part of the data which I have studied, that the radial and tangential velocities are connected by the relation $\frac{v}{u}=Cu$, a constant, and the motion parallel to, or along the axis, is connected by the relationship $\frac{v^2}{w}=Cw$, constant, which is according to our third assumption that the radial velocity is directly proportional to the tangential, while the axial velocity is proportional to the *square* of the tangential velocity.

The experimental data on which Taylor and Webb have based their discussion in comparing the mathematical theory with the results obtained from some actual measurements of the rate of decay of eddies in a wind tunnel, are contained in a paper by Relf and Lavender (**1a**) on an investigation of the effect of upwind disturbances upon the forces measured on models placed in the channel.

Their experiments were made with a screen of vertical cords of ¼ inch (0.6 cm.) diameter spaced near each other across the tunnel, for the purpose of learning just what effect the eddies set up by such a screen might have upon the forces measured on models at various distances behind it.

They found that the time taken for an eddy of a given initial size to die down to a given fraction of its original intensity is constant, and the distance in which a given decrease in intensity results should therefore be proportional to the wind speed.

My own observations have clearly substantiated the first part of this general result, but they have failed to give quite the same indication in regard to their assertion that a given decrease in intensity should be in proportion to a definite fraction of the wind speed, since, as already mentioned, no direct linear relation could be found between the two in the case of the experiments described in an earlier part of this paper.

The results obtained from these observations have appeared to indicate that the tangential velocity of the eddy motion does not increase proportionally as the velocity of the mean motion increases. It is relatively higher at the lower wind speeds, so that we might say, for instance, that the tangential velocity increases more rapidly in proportion to an increase of from 10 to 15 meters per second in the mean motion than it does to an increase of from 20 to 25 meters per second.

In one case where the mean relative velocity was a little over 40 feet per second, the tangential velocity was 3.15 m.p.s., while on the other hand, when the mean velocity was around 25 m.p.s., the tangential velocity was found to be only 33.3 per cent greater, or about 4.2 meters per second.

In Relf and Lavender's experiments, for a wind speed of 6.1 m. p. s., the required distance of the screen from the model to reduce the effect to half value was 22 inches. At 12.2 m. p. s. the distance in which the effect was halved was 38 inches, and at 18.3 m. p. s. it was 48 inches.

The screen ceased to have any effect on the model when the distance between them was increased to about 7 feet, so that at this distance the result was practically the same as that for a clear channel.

In other words, this would mean that if we reversed the process and assumed the screen to move through still air at a speed of 12.2 m. p. s. (40 feet per second) the eddies produced at a given point will have become entirely dissipated by the time it has moved about 7 feet.

$$\text{Now } \frac{v}{l}=\frac{40}{7}=5.7 \text{ and therefore } \frac{1}{v/l}=0.1754$$

or practically 0.18 second.

Calling this T_2 and let $T_1=9$ seconds (the time required for an eddy of 2 meters radius to die down from an initial tangential velocity of $v_1=3.15$ meters per second at the same wind speed of 40 feet per second), then the tangential velocity v_2 of an eddy of 0.6 cm. radius is

$$(6)\quad v_2=\frac{v_1}{T_1/T_2}=0.063 \text{ meters per second,}$$

which is about one-half of 1 per cent of the wind speed.

Hence

$$\frac{0.063\times 1.20}{0.15}\times 0.3572=0.18 \text{ second.}$$

No doubt the tornado is the most conspicuous and well defined of all forms of eddy motion in the atmosphere, and it is of interest to give some thought to the question of how long one of these violent storms would continue to exist as a free eddy if the source of energy by which it is originated and maintained would suddenly cease to operate.

The maximum tangential velocity of the air near the center of a fully developed tornado has been variously estimated at around 200 to 225 meters per second.

Taking the maximum velocity as 200 m. p. s., the time required for it to become entirely dissipated is found from (5) to be equal to

$$\tau=\frac{200\times 1.20}{0.15}\times 0.3572=571.52 \text{ seconds,}$$

or about 9.5 minutes.

This is approximately what we might expect from actual observation, considering the suddenness with which these storms develop, and the equal suddenness with which they die away and disappear when conditions are no longer favorable for their maintenance.

We may therefore be justified in the conclusion that the source of energy of the tornado [1] is not by any means localized within the area of the disturbance itself, but that it probably has its origin at an altitude of at least one or two thousand meters, and that it is due to some large-scale process of a sudden equalization of the potential differences between oppositely directed air streams of widely different temperatures within that sector of the general low-pressure system where tornadoes are usually known to occur, especially within the southeast or southwest quadrant of a rather intense LOW where a deep current of cold air banked in the rear is being suddenly released and allowed to run under or over the warm current at a comparatively sharp angle so as to set up a very strong mechanical convection and to bring the two oppositely directed air streams into close contact while having a great difference in velocity, temperature, and humidity, etc.

[1] Cf. The Tornado, by W. J. Humphreys, *54*: 501-503. Ed.

ON THE PRODUCTION OF TURBULENCE IN THE ATMOSPHERE

In an earlier part of this paper it was explained how eddies are produced by the relative motion between a body and a fluid as the fluid is fed backward and released from the rear terminal of the boundary layer.

The conception of a boundary layer forming the region of transition between a solid body and the surrounding air is of the utmost importance, not only from an aerodynamical standpoint but also in its relation to certain meteorological problems as well.

It is not at all necessary that the fluid movement should be brought to some sort of an abrupt corner in order that eddies may originate from within the boundary layer, as it is entirely possible for eddy motion to be set up after a fluid is made to move for a certain time over a relatively smooth flat surface, and in fact eddies are continually being formed in this way as the wind blows over an open, level field.

In most cases, however, eddies very near the earth's surface are formed by various obstructions to the wind movement, such as buildings, trees, fences, etc., and by variations in surface topography; but these eddies are sensibly stationary, so that in a steady wind any light material, such as dust, snow, or chaff, etc., which may be carried along in the air movement is seen to always follow through the same trajectory, or path of motion, in passing any particular one of these obstructions.

It is quite convincing of this to notice how snow is carried up into wavy drifts over an open field by the action of the wind along the surface, especially when there is already several inches of snow on the ground, and this is scoured by a brisk wind while the temperature is falling, so that the snow is packed considerably as it drifts.

The same thing will also be noticed in the case of snow drifts formed around any stationary object which happens to be in the path of the wind.

It is this process of eddy formation near any relatively stationary boundary of the fluid which is supposed to be the active agency by which momentum is transferred from the fluid to the solid surface, or from one fluid layer to another, where there is a difference in velocity between them.

The intensity of the eddy "circulation" will be proportional to the rate of increase of velocity in a direction normal to the boundary surface.

It would be a matter of considerable interest to know just what relation may exist between the scale of the mean motion—that is, the height or thickness of the fluid strata within which eddies are being produced—and the velocity, or rather the difference in the mean velocity of flow of these separate layers, and the size and intensity of the eddies themselves.

This represents rather a difficult problem because there is so little experimental data available which might be taken even as a partial basis for determining this relation.

In Figure 5 is shown the velocity gradients for different rates of relative motion between two layers of height, h_1 and h_2, moving in opposite directions.

The viscous drag between the two layers brings the relative velocity to zero at some point midway between them, which is here taken to be the boundary of separation of the two currents.

Besides their difference in velocity and direction they are also likely to be of different temperatures and water vapor content.

It frequently happens that where two oppositely moving currents are thus brought into close proximity to each other the velocity gradients reach a certain critical value for those particular layers of fluid, and the stresses developed within them reach the point where stability is no longer possible for laminary flow, and possibly through some local variation in pressure a rotation of a small portion of the fluid will be suddently set up near the boundary between them, which quickly develops into an eddy circulation with an irrotational field around the nuclear axis in accordance with the requirements for the conservation of angular momentum about this central axis.

There is very little possibility indeed of two such relatively moving currents ever being exactly equal to each other and opposite in direction, and even then it would hardly be dynamically possible for the center of rotation to remain fixed in space.

What usually occurs in practically every instance where eddies are formed at the boundary of separation between two currents is for the eddies to be carried along in one or the other, depending upon which is the stronger of the two streams or fluid, so that their tangential velocities about the axis of each individual eddy is combined with a sort of rolling motion along a line somewhere near the boundary.

Some general idea as to the pattern of the streamlines about the moving axis is illustrated here in the diagram, Figure 6.

The actual fluid motion takes place through a process somewhat analogous to the principle of caterpillar traction, where the rolling wheel lays down its own track as it moves along.

In the same way, the fluid flowing round on the forward moving side of the eddy loses much of its velocity of translation as it turns around to the reverse side of the center, as here it is coming into contact with the relatively stationary fluid forming the neighboring portion of the other layer, and in fact a considerable part of one strata may be deposited and mixed with the other in this way as the fluid from more distant regions (higher or lower as the case may be) is drawn in toward the center as it moves along near the boundary.

The temperature and water vapor content of the air thus drawn in would probably vary from time to time, and as would also the amount of condensation taking place, so that we might in this way account for the various forms of clouds observed at different levels where one layer of air may be running over or under another with sufficient relative velocity to cause the formation of eddies.

Now as to the possible relation between the scale of the mean motion and the size and intensity of the eddies produced by it, we may reasonably suppose that the principle which governs this relation in regard to the eddies produced by relatively moving currents in the free atmosphere, is the same as that already mentioned for those formed in the turbulent wake following the relative motion of solid bodies through a fluid otherwise at rest.

Returning to Figure 6, where h_1 and h_2 is the height or thickness of two relatively moving fluid strata within which the eddy motion takes place, u_1 and u_2 are the maximum velocities along the two sides in the directions of x_1 and x_2, respectively; S is the surface of separation between them, where the velocity is supposed to be zero; v is the tangential velocity of the eddy, and V is its velocity of translation along the line connecting the points q_1 and q_2.

It is quite evident that the radius r and the tangential velocity of the eddy must therefore be some function of the height h_1 h_2 and the mean velocity u_1 u_2, and perhaps also the translational velocity V.

If h_1 h_2 are decreased by a certain amount, the radius decreases, and at the same time v would necessarily have to increase if the mean relative velocity between the two streams remained constant; but if this velocity of the mean motion increased without any change in h_1 or h_2, then the radius of the eddy would remain the same and the tangential velocity would not increase by a proportional amount on account of the increase in V. The tangential velocity and also, consequently, the angular momentum measured from the forward moving side of the eddy at any point along a line normal to the direction of q_2 would be equal to $u-V$.

This also applies particularly to the case where one of the fluid layers involved in the relative motion happens to be the comparatively stationary boundary layer immediately in contact with the earth's surface. It is in this region that we find the greatest increase in velocity compared to the least difference in height above the surface, and therefore the eddies are of smaller radius and of greater intensity here than anywhere else.

The combined effect of the rolling motion and the transverse velocity around the moving axes is to modify, or reduce to a minimum, the velocity gradients set up between the stationary boundary layer and the maximum velocity attained by the wind at a given height above the surface.

On account of the fact that this rolling movement of the eddies introduces a more complex relation between the velocity of mean motion and the transverse velocities, referred to these moving axes, it may be that a given increase or decrease in the mean velocity of flow or a certain change in the rate of increase of wind velocity with height does not always result in a proportional difference in the amount of momentum being transferred from one atmospheric layer to another. This is a phase of the problem which will require considerably more study and investigation from an observational standpoint before it can be expected to give entirely consistent and satisfactory results from its analytical treatment.

But while the general problem of turbulent motion in the atmosphere is still far from being thoroughly understood, and there is at present but little hope of its complete solution at any time within the near future, the quantitative investigation of the way in which the wind varies with height and the average rate at which momentum is transferred from one atmospheric layer to another (whatever may be the actual mechanism of these processes) has already reached the point where it has yielded some extremely valuable results. It has been made the subject of a number of very important contributions by Taylor (**10**), Hesselberg and Sverdrup (**11**), Åkerblom (**12**), Whipple (**13**), Brunt (**14**), and others.

In the case of turbulent motion at the surface, it has been found that the relation of wind velocity to height follows approximately a logarithmic law, and when there is no change of gradient, or of eddy viscosity with height, the logarithmic relation can be represented by the addition of a vector to the line representing the gradient wind, where the direction is along or tangential to the isobar, such that this added vector represents the actual wind, both in magnitude and direction, and its point sweeps around an equiangular spiral of 45°, making equal steps of rotation for equal steps of descent from the undisturbed wind to the surface.

A very clear explanation of the law of variation of wind with height has been given in a recent paper by Humphreys (**15**).

The frictional force R, acting upon unit vloume of the atmosphere, may be measured by the amount of horizontal momentum lost per unit of time. According to Taylor, the components of R along the axes of x and y, are

$$K\rho\frac{d^2u}{dz^2} \text{ and } K\rho\frac{d^2v}{dz^2}, \text{ respectively,}$$

R being a function of the height z.

If we let F be the frictional force per unit surface acting at any place, then it may be shown that

$$R=-\frac{dF}{dz}=K\rho\frac{d^2V}{dz^2}$$

where V is the wind velocity.

Taylor denotes $\frac{1}{2}\,u\rho h$, which he calls the eddy viscosity, by the letter μ, so that the rate of transfer of momentum downward may be written $\mu\frac{\delta V}{\delta z}$.

It follows from this that the rate of transfer downward is

$$\sqrt{2}\mu BR=K\rho V^2.$$

If the thickness of the layer is dz, then the difference between the net flow of momentum into the layer from above and the flow out of it from below is

$$\sqrt{2}\mu B\frac{\delta R}{\delta z}dz.$$

Thus the additional momentum gained in unit time by unit volume of air on account of the eddies is $2B^2\mu R$. Where B is a constant and is independent of Z.

There are two assumptions in connection with this theory which it seems are still to be regarded as lacking in definite proof as to their validity. One of these involves the question as to whether K can always be taken as constant throughout the height considered, and the other is whether B also is constant with height.

Some discrepancy has been found between the observed and theoretical values for the angle between the frictional force R and the reversed wind direction, and apparently this can not be explained on the assumption that B and K are constant with height.

It is possible, too, that the results obtained in a given case may be affected in some way by convectional activity, or by some other factor which has not yet been thoroughly investigated.

There remains to be mentioned one other important relation, derived from the study of the rate of decay of eddies, which may be of some interest in connection with this problem of the transference of momentum between different fluid layers in the atmosphere.

Suppose we let v represent the difference in the mean velocity u_1 and u_2 of two layers at heights z_1 and z_2, respectively, and where this difference in height dz is not too great, we may let ρ be the average density of the air within or between these two layers.

Now for a given initial velocity, and in the absence of any forces except that due to the momentum at the instant $t=t_o$, the time required for the motion to die away is, according to (5),

$$\tau=\frac{\rho v}{v}\times .3572$$

and from this it is found that the rate at which momentum is transferred from one layer to another is

$$(7)\quad Cm=\frac{\rho v/v}{\tau}=2.8$$

So that where the motion is dying down the amount of momentum transferred in unit time is constant and equal to $Cm=2.8\times10^5$ in c. g. s. units, and therefore under steady motion it would require an acceleration equal to the above amount in order to balance the loss due to eddy viscosity.

It will be noticed that Cm is in this case on an average about 3.5 times as large as the mean value for K as determined in the work of Taylor and Åkerblom, and also since its value depends on that of C_T, it is probably not constant with height, but I have not yet had an opportunity to attempt to check or compare this with observational data.

CONCLUSION

As explained at the beginning of this paper, with the limited facilities available it was impossible to attain a sufficient degree of accuracy in the collection of these data for the results to approach what might be considered a final form of exactness. They were presented here only as a first approximation to the solution of the problem, and the discussion which followed was intended to give some idea as to the important bearing which this special subject has in connection with a number of other major problems more or less related to it.

It still remains, however, for much more experimental and observational work to be done, under conditions where it will be possible to make precise measurements of the rate of decay of eddies of various dimensions and at a number of different velocities of mean wind movement.

It is suggested that wind-tunnel tests be made with a delicate Pitot tube, or a hot-wire anemometer, to determine the rate at which the fluid is released from the boundary layer at the rear contour of bodies of various shapes and sizes, and for different velocities of flow; and then the distance to which the effect of a given disturbance, or one of known intensity, is carried, and the time required for it to die away for any given velocity of flow.

A series of tests of this character in a variable-density wind channel should also furnish some valuable information on the variation of C_T with altitude.

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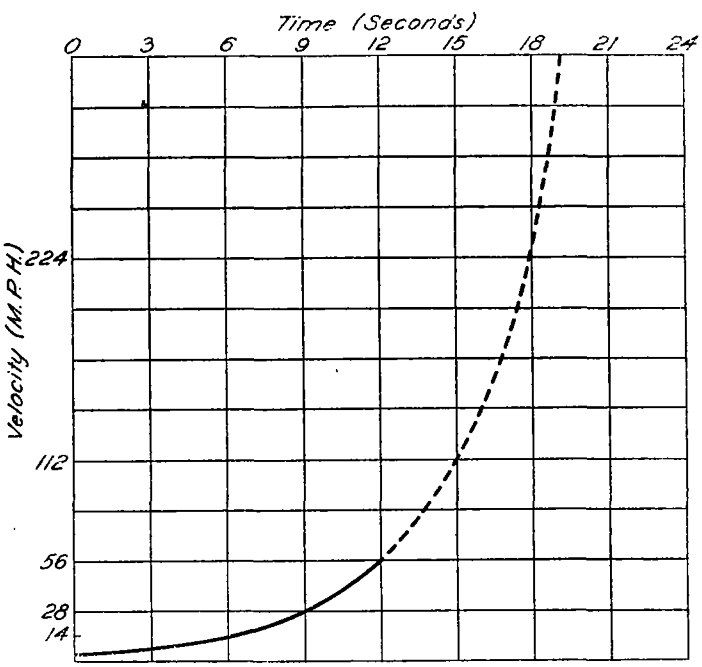

FIG. 1.—Curve showing relation between the speed of the train and the time rate of decay of the eddies produced by it

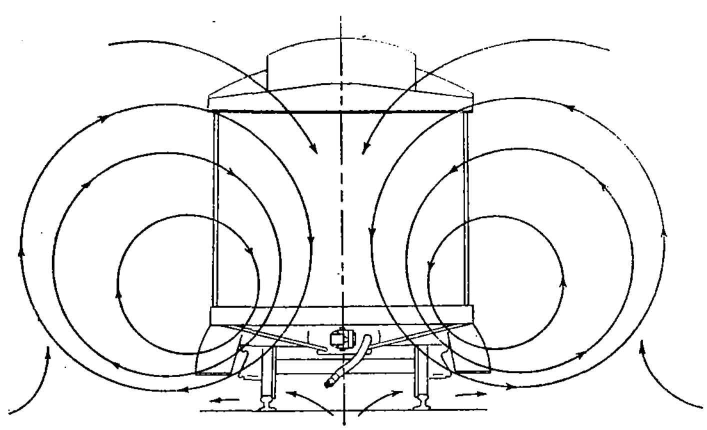

FIG. 2.—Two-dimensional diagram of the flow pattern of the eddies produced by a moving train

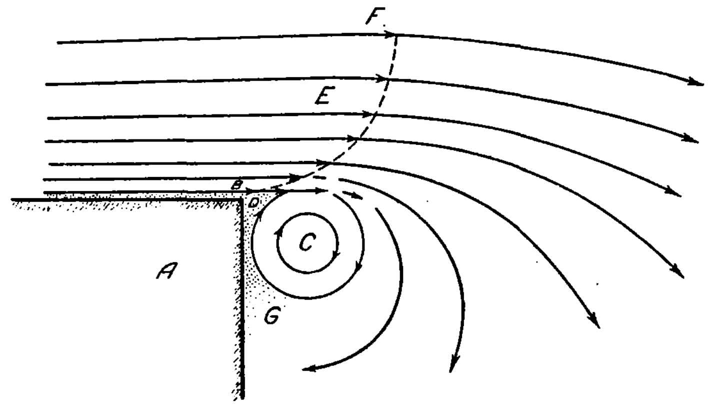

FIG. 3.—Deflection of air current which passes along a stationary wall which suddenly breaks off at an angle. It is assumed that the motion is steady and two-dimensional

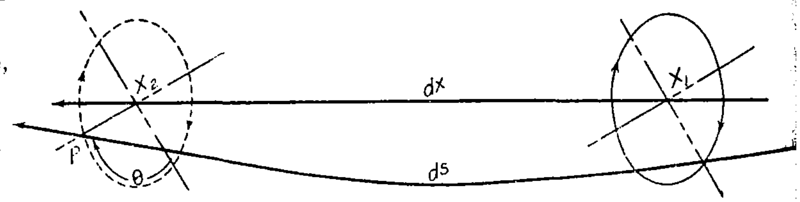

FIG. 4.—Tangential velocity of the fluid following the helical path of motion mentioned on page 269 of this article

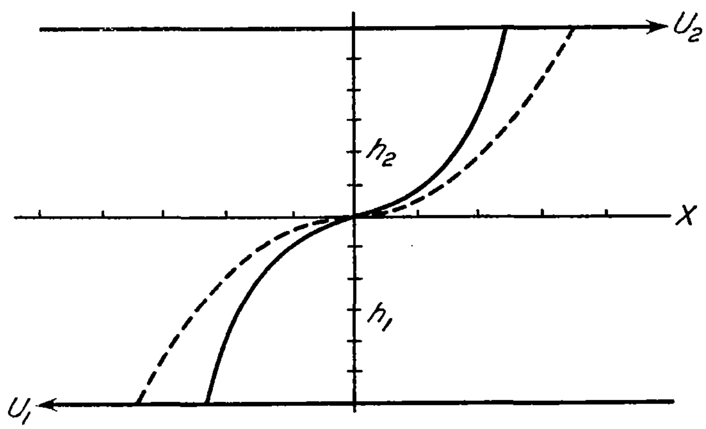

FIG. 5.—Velocity gradients for different rates of relative motion between two layers of height, h_1 and h_2, moving in opposite directions

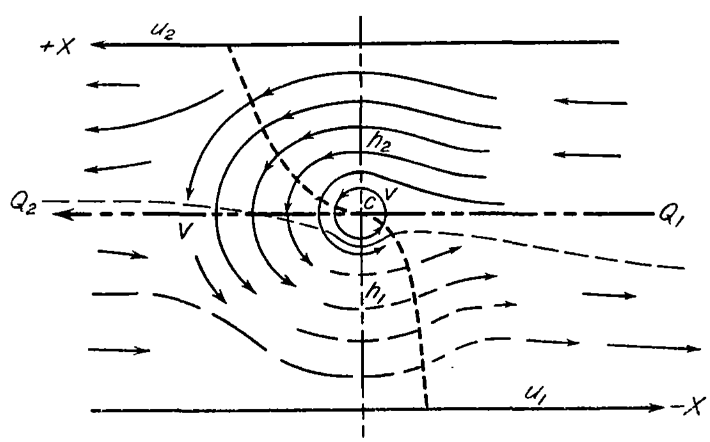

FIG. 6.—Pattern of streamlines about the moving axis